Discipline: Operations (OM / OR)

1 Language

The course will be held in English on demand by any participant, otherwise in German.

2 Title

Probabilistic Models and Stochastic Programming

3 Lecturer

Prof. Dr. Stefan Helber (Leibniz University Hannover)

https://www.prod.uni-hannover.de/helber.html

stefan.helber@prod.uni-hannover.de

4 Date and Location

February 25-28, 2019, Leibniz University Hannover

5 Course Description

5.1 Abstract and Learning Objectives

The course covers the basic elements of i) Markovian models of stochastic systems and ii) Markovian decision processes as well as iii) basic elements of stochastic programming using scenario techniques. In this course, participants will learn how to construct and use these particular classes of probabilistic models of systems and decision processes or situations. The defining feature of the Markovian models is the memoryless property of the underlying stochastic processes. It essentially states that the future behavior of a system or decision process depends only on its current state, but not its previous history. The participants will learn why and how this often makes it possible to determine the probabilities of the different system states and how these probabilities can then be used to determine performance measures of the system or to assign economic values to decisions made in an uncertain environment.

5.2 Content

We will start by describing the defining features and variants of stochastic processes, focusing on those processes with a discrete state space. We will initially cover Markovian processes in discrete time as their modeling and analysis happens to be mathematically straightforward, clear and illustrative. On this basis, we will then address Markovian processes in continuous time, the so-called Markov Chains. Their modeling and analysis is mathematically somewhat more abstract. However, those models can be considered to be more flexible and general. We will describe Poisson processes, study the famous memoryless property of the exponential distribution and derive the necessary and sufficient balance equations for the steady-state analysis of Markov Chains in continuous time. The participants will install and use *Scilab*, an free and open-source software for numerical computation (www.scilab.org), on their machines to perform numerical experiments and solve small exercises.

Based on this foundation, we will then address the analysis of Markovian queuing systems and show how to determine the probability of having a given number of customers or jobs in the system or the queue. After introducing and explaining Little's Law, we will use it to determine expected waiting or cycle times.

On this foundation of Markov chains and as an example of how to apply the methodology, we will also study the analysis and the behavior of basic flow line models consisting of two machines with a buffer of limited size in between the two machines. Randomness in those systems can affect processing times, times to failure and times to repair. Students will be given modeling assignments to eventually set up their own programs in Scilab to determine performance measures of a *given* Markovian service or production system.

In the next step, we will aim at numerical methods to provide *decision support* in a stochastic environment. We will start with two-stage decision processes where in the first stage a decision, e.g., about at production quantity, is made based on probabilistic distribution information about some other quantity, e.g., product demand, that will only be known *after* that first-stage decision has been implemented. Using the famous newsvendor problem as a starting point and then proceeding to facility location and capacity decision problems involving discrete and binary decision variables, we will show how to model those problems using the GAMS modeling language. We will embed (conceptually rather simple) simulations within (mixed-integer) linear programming models that we will solve numerically in order to determine solutions that reflect a decision-maker's risk attitude. We will briefly touch variance reduction methods to reduce the required sample size while still leading to the desired degree of precision.

The final part of the course will be devoted to stochastic (dynamic) programming, also known as stochastic decision processes. Here we will close the circle by returning to Markovian models again, now in the context of decision making. Starting with a finite-horizon deterministic setting, we will treat the basic version of Bellman's recursion equation and explain its usage to determine an optimal course of action in a multi-stage decision situation. On this foundation, we will next consider the stochastic case of a finite-horizon Markovian Decision Process in discrete time, explain the optimality equations and their solution via the backward induction algorithm. Finally, we will consider the infinite-horizon, continuous time case of discounted Markov decision processes and the value iteration algorithm to determine policies that are optimal in expectation.

Ample opportunity will be given to work in a supervised manner on small modeling and analysis assignments and to implement those using Scilab or GAMS.

We will furthermore discuss current research projects of the participants that involve probabilistic modeling. Participants are therefore invited to submit proposals for 30-minute presentations on their own projects if those exhibit important elements of stochasticity. Students are expected to elaborate on those stochastic elements, why they matter for the respective performance analysis or decision support problem and how they are contemplating to deal with them. We will then discuss the used or proposed modeling approaches as well as potential alternatives.

5.3 Content

Day I: (25.02.2019)

10:00-10:30	Arrival of participants, reception, check-in and introduction
10:30-12:30	Discrete state stochastic processes, Markovian processes in discrete time, classification of discrete time Markov chains, transition matrix, steady state probabilities
12:30-13:30	Lunch break
13:30-15:00	Introduction to Scilab, experimentation with Scilab codes to analyze discrete-state discrete-time Markov processes
15:00-15:15	Coffee break
15:15-16:45	Modeling and analysis exercises of discrete time Markov chain models using Scilab
16:45-17:00	Coffee break
17:00-18:00	Presentation and discussion of a participant's research project involving probabilistic modeling

Day II: (26.02.2019)

9:00-10:30	Markovian processes is continuous time, Poisson process, exponential distribution, memoryless property of the exponential distribution
10:30-11:00	Coffee break
11:30-12:30	Steady-state balance equations and analysis of continuous-time Markov chains
12:30-13:30	Lunch break
13:30-15:00	Modeling exercises of continuous time Markov chains using Scilab: Analysis of one- and two stage production system with limited buffer capacity and reliable or unreliable machines
15:00-15:15	Coffee break
15:15-16:15	Presentation and discussion of a participant's research project involving probabilistic modeling
16:15-16:30	Coffee break
16:30-17:30	Presentation and discussion of a participant's research project involving probabilistic modeling

Day III: (27.02.2019)

9:00-10:30	Markovian birth-and-death processes in continuous time, steady-state analysis of the M/M/1 queueing system, Little's law, analytical results for performance measures
10:30-11:00	Coffee break
11:00-12:00	Analysis of M/M/C and M/M/1/K queuing models, structural behavior of queuing systems
12:00-13:00	Lunch break
13:00-14:00	Combining optimization and simulation modeling via two-stage (mixed-integer) linear programs using scenarios, variance reduction techniques, experimentation using GAMS models, exercises
14:00-14:15	Coffee break
14:15-15:45	Modeling exercises involving two-stage programs and scenarios
15:45-16:15	Coffee break
16:15-17:15	Presentation and discussion of a participant's research project involving probabilistic modeling

Day IV: (28.02.2019)

9:00-10:30	Multi-stage decision making in a finite-horizon deterministic setting, state and action space, transitions, Bellman's equation and the backward induction principle, modeling exercises
10:30-11:00	Coffee break
11:00-12:30	Markovian decision processes with finite horizon, backward induction, infinite-horizon continuous time case, discounted Markovian decision processes, value iteration algorithm, modeling exercises
12:30-13:30	Lunch break
13:30-14:30	Presentation and discussion of a participant's research project involving probabilistic modeling
14:30-14:45	Coffee break
14:45-16:15	In-class exam (optional)
16:15-16:30	Wrap-up and feedback

5.4 Course format

The course will consist of a mixture of lectures, exercises (mostly involving Scilab or GAMS programs or models) and seminar elements. The seminar elements will be devoted to current research projects of the participants and the probabilistic models they are attempting to analyze. The course will be held in English on demand.

6 Preparation and Literature

6.1 Prerequisites

Master-level education in Business, Economics, Computer Science, Engineering or a related field. Students are expected to be familiar with and able to use calculus, linear algebra, random variables including their probability distributions and their moments. Students should feel comfortable to use a computer to perform a quantitative analysis

6.2 Essential Reading Material

Relevant chapters (see timetable and content) of the following books:

Stewart, William J. (2009): Probability, Markov Chains, Queues, and Simulation. The Mathematical Basis of Performance Modeling, Princeton University Press

Puterman, Martin L. (2005): Markov Decision Processes: Discrete Stochastic Dynamic Programming, Wiley

6.3 Additional Reading Material

Students will be provided with slides as well as selected Scilab and GAMS files.

6.4 To prepare

Participants are strongly encouraged to contact the instructor and to propose a presentation of a research project of their own related to stochastic modeling to fill one of the five student's project presentation slots. Please contact the instructor in advance in order to schedule this talk. If you are selected to present, please prepare a presentation for 30 minutes based on at most 20 slides. After each such presentation, 30 additional minutes will be devoted to questions and discussion. Alternatively, some students can present and discuss a relevant research paper from the literature with the instructor's consent in the unlikely case that fewer than five appropriate student's projects are proposed.

7 Administration

7.1 Max. number of participants

The number of participants is limited to 20.

7.2 Assignments

Students will be given assignments/exercises to be solved during the course.

7.3 Exam

A 90-minute in-class exam will be offered at day IV (optional). One third of the grade will reflect the intensity and quality of classroom participation and exercise work. For those students who give a presentation, the grades for the presentation and for the final exam also have a weight of one third each. For students not giving a presentation, the final exam has a weight of two thirds.

7.4 Credits

The course (including the exam) is eligible for 6 ECTS.

8 Working Hours

Working Hours	**Hours**
Preparation: Review of basic elements of probability theory *(Concept of random experiment, concept of probability, discrete random variables, continuous random variables, expectation and higher moments of random variables, random variables in higher dimensions, concept of conditional probability)*	20 h
Preparation: Review of basic linear algebra *(Vectors, matrices, vector and matrix addition and multiplication, systems of linear equations, invertibility of a matrix, inverse of a matrix, solution of a system of linear equations)*	20 h
Preparation: Review of software for numerical computation *(Installation of Matlab or Scilab on own mobile computing device, numerical solution of systems of equation, plotting of functions, programming of modular software with global and local variables using functions and subroutines)*	20 h
Preparation: Reading of the course-relevant pats of the books by Stewart and Puterman	40 h
Preparation of a presentation on a problem involving probabilistic modeling	40 h
Classroom work including lectures and exercises	30 h
Exam preparation and exam	10 h
TOTAL	**180 h**
ECTS: 6	